Math 8-Adv

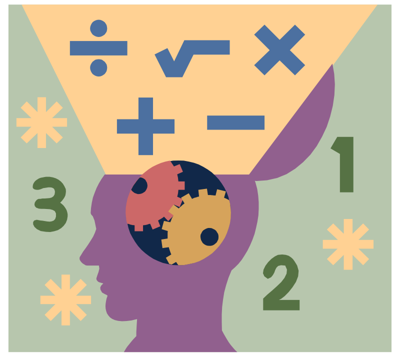
Colonial School District

Summer Math Packet

The concepts included in this packet will help reinforce key skills your child has encountered in math this year. Please encourage them to complete as many activities as possible as it will lead to greater success next year. The answer key to this packet is available on the district website (www.colonialsd.org).

Colonial
SCHOOL DISTRICT

Promoting a Culture of Collaboration, Innovation and Inspiration



June

Dear Parents/Guardians,

First, we would like to thank you for all of the additional support you offer at home. Education is a true partnership between school and family that is essential to a child's success.

As this school year comes to a close, we wanted to again encourage you to continue to reinforce and foster the mathematical skills and practices that have been developed this year by scheduling time for your child to work through this summer math packet. The activities were selected by our grade level experts with the key mathematical concepts of the school year in mind. The ultimate goal is to reinforce and strengthen the skills that will serve as building blocks for future learning.

Wishing you a relaxing, yet exciting, math-filled summer!

Sincerely,

The Curriculum Department

Serving the students of Conshohocken, Plymouth and Whitemarsh

230 Flourtown Road, Plymouth Meeting, PA 19462 – Phone (610) 834-1670 – Fax (610) 834-7535 – www.colonialsd.org

Equations

Two-step Equations

RULE	EXAMPLE
1. First, undo addition or subtraction. 2. Then, undo multiplication or division. 3. Check your answer by replacing the variable with the solution.	$3x - 2 = 13$ $\underline{+2} \quad \underline{+2}$ $\frac{3x}{3} = \frac{15}{3}$ $x = 5$ ✓ $3 \times 5 - 2$ $15 - 2 = 13$

Solve.

1. $6d - 3 = 32$

2. $\frac{x}{5} + 2 = 6$

3. $2y + 7 = 15$

4. $\frac{b}{7} - 13 = 23$

5. $-5y + 9 = 24$

6. $\frac{f}{8} - 3 = -27$

Equations

Equations with Variables on Both Sides

RULE	EXAMPLE
1. Eliminate the variable from one side of the equation using inverse operations. 2. Undo addition or subtraction. 3. Then, undo multiplication or division. 4. Check your answer by replacing the variable with the solution.	$8x - 3 = 6x + 1$ $-6x \quad -6x$ $2x - 3 = 1$ $+3 \quad +3$ $\frac{2x}{2} = \frac{4}{2}$ $x = 2$ ✓ $8 \times 2 - 3 = 6 \times 2 + 1$ $16 - 3 = 12 + 1$ $13 = 13$

Solve.

1. $3k + 10 = 2k - 21$

2. $x - 4 = 6x - 19$

3. $18 + 4p = 6p + 11$

4. $11h - 14 = 7 + 14h$

5. $-3p + 8 = 2p - 2$

6. $-t + 10 = t + 4$

Name ______________________ Date ______________

Reteaching Worksheet 7-5

Solving Multi-Step Equations

When an equation includes parentheses, you often must first simplify the equation before you can solve it. Compare these examples.

$$\begin{aligned} 4(x - 6) &= -44 + 3x \\ 4x - 24 &= -44 + 3x \\ 4x &= -20 + 3x \\ x &= -20 \end{aligned}$$

$$\begin{aligned} 3(2x - 5x) &= x + 45 \\ 3(-3x) &= x + 45 \\ -9x &= x + 45 \\ -10x &= 45 \\ x &= -4.5 \end{aligned}$$

1. Check the left example.

2. Check the right example.

3. Compare the first step in the two examples. What operations are used?

4. In the left example, what happens if you first divide both sides by 4?

Solve each equation. Check your solution.

5. $6(4x - x) = -36$

6. $6(4x - x) = 12x - 36$

7. $-2(-3x + 6x) = 30$

8. $-2(-3x + 6x) = 30 - x$

9. $-8(x - 20) = -96$

10. $-8(x - 20) = -96 - 4x$

11. $75 = 5(-4 + 2x)$

12. $75 - 9x = 5(-4 + 2x)$

13. $2.5 = -0.5(x - 1.5)$

14. $2x + 2.5 = -0.5(x - 1.5)$

Name ______________________________ Date ______________

Reteaching Worksheet 3-7

Solving Inequalities: Adding and Subtracting

An inequality is a mathematical sentence that contains one of these symbols: $<$, $>$, $\leq$, $\geq$, or $\neq$. The meaning of each of these symbols is given in the table at the right.

The same steps used to solve equations are used to solve inequalities.

Symbol	Meaning
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
$\neq$	is not equal to

Example: Solve the inequality $x + 6 > 11$.

$$x + 6 > 11$$

$$x + 6 - 6 > 11 - 6 \quad \text{Subtract 6 from each side.}$$

$$x > 5$$

Check: To check the solution, replace x with any number greater than 5. For example, use 8.

$$x + 6 > 11$$

$$8 + 6 \overset{?}{>} 11$$

$$14 > 11 \checkmark$$

The solution is any number greater than 5.

Solve each inequality. Check your solution.

1. $x + 41 < 6$
2. $x + (-4) < 20$
3. $x - 7 < -10$
4. $x - 75 > 27$
5. $x - (-5) > 21$
6. $x + 54 > -96$
7. $x + (-12) > 20$
8. $x - 104 < 75$
9. $x + 7 > 15$
10. $x - 32 < -12$
11. $x - 72 > -136$
12. $x - (-92) < 65$

Glencoe Division, Macmillan/McGraw-Hill

Name ______________________ Date ______________

Reteaching Worksheet 3-8

Solving Inequalities: Multiplying and Dividing

When you multiply or divide each side of an inequality by a positive number, you get a new inequality with the same solutions.

$$3h < -12$$
$$3h \div 3 < -12 \div 3$$
$$h < -4$$

$$\frac{h}{5} > 10$$
$$\frac{h}{5} \cdot 5 > 10 \cdot 5$$
$$h > 50$$

When you multiply or divide each side by a negative number, you must reverse the inequality symbol. Otherwise, the new inequality will not have the same solutions.

$$-3h < -12$$
$$-3h \div (-3) > -12 \div (-3)$$
$$h > 4$$

$$\frac{h}{-5} > 10$$
$$\frac{h}{-5} \cdot (-5) < 10 \cdot (-5)$$
$$h < -50$$

Do the two inequalities have the same solutions? Write yes or no.

1. $2x < 14$
   $x > 7$
2. $-x < 0$
   $x > 0$
3. $3x < 9$
   $x < 3$
4. $-5x > 0$
   $x > 0$
5. $-4x < 4$
   $x > -1$
6. $-3x > -3$
   $x > 1$

Solve each inequality. Check your solution.

7. $7x < 84$
8. $9x > 81$
9. $\frac{h}{3} < -10$
10. $6p < 12$
11. $\frac{h}{4} > -7$
12. $0 > -5c$
13. $-2d > 4$
14. $-2d > -4$
15. $-2d < -4$
16. $\frac{a}{-3} < 9$
17. $\frac{a}{-3} > -9$
18. $\frac{a}{3} < -9$

Name ______________________ Class ______________ Date ______________

Review 67

OBJECTIVE: Solving systems of linear equations by substitution

MATERIALS: None

Example

Solve using substitution.

$$-4x + y = -13$$
$$x - 1 = y$$

$y = 4x - 13$ ← Rewrite each equation in the form $y = mx + b$.

$y = x - 1$

$y = \boxed{4x - 13}$ ← Circle the sides of the equations that do not contain y.

$y = \boxed{x - 1}$

$4x - 13 = x - 1$ ← Since both circled parts equal y, they are equal to each other.

$3x = 12$ ← Solve for x.

$x = 4$

$y = 4x - 13$ ← Substitute 4 for x in either equation. Solve for y.

$y = 4(4) - 13$

$y = 3$

The solution is (4, 3).

Check to see whether (4, 3) makes both equations true. If it doesn't, then the system has no solution.

$-4(4) + 3 \stackrel{?}{=} -13$ $\qquad$ $4 - 1 \stackrel{?}{=} 3$

$-16 + 3 \stackrel{?}{=} -13$ $\qquad$ $3 = 3$ ✓

$-13 = -13$ ✓

Exercises

Solve each system using substitution. Check your solution.

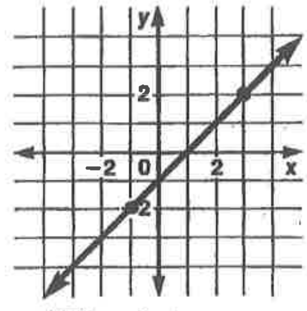
1. $-3x + y = -2$
   $y = x + 6$
2. $y + 4 = x$
   $-2x + y = 8$
3. $y - 2 = x$
   $-x = y$
4. $6y + 4x = 12$
   $-6x + y = -8$
5. $3x + y = 5$
   $2x - 5y = 9$
6. $x + 4y = 5$
   $4x - 2y = 11$
7. $2y - 3x = 4$
   $x = -2$
8. $3y + x = -1$
   $x = -3y$
9. $2x + y = -1$
   $6x = -3y - 3$

Name ______________________________ Date ______________

Reteaching Worksheet 8-6

Graphing Equations

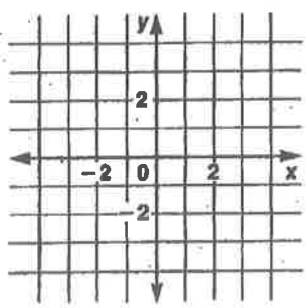
An equation has many ordered pairs of values that are solutions. For example, four ordered pairs for the equation $y = x - 1$ are (3, 2), (0, −1), (2, 1), and (−1, −2). There are too many to name so a picture is drawn of them. This picture is called a graph of the equation. The graph of $y = x - 1$ is the line drawn on the coordinate system at the right.



Find three ordered pairs that satisfy each equation. Graph each ordered pair. Draw a line through the points.

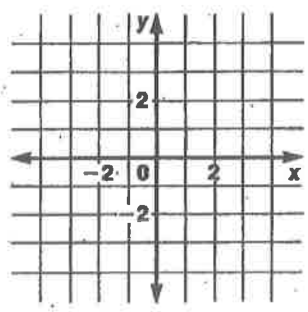
1. $y = x$

x	y
−2	−2
0	
3	



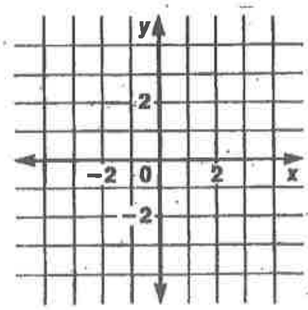
2. $y = x + 1$

x	y
−1	
0	
3	



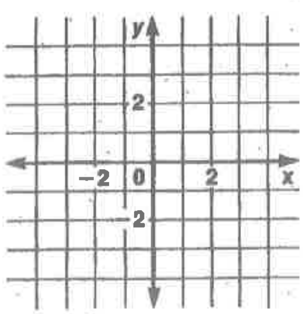
3. $y = x + 2$

x	y
0	
1	
2	

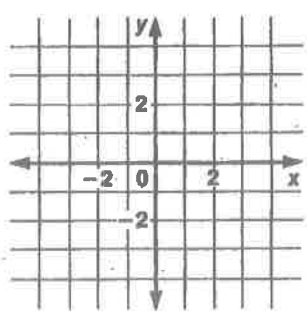


Graph each equation.

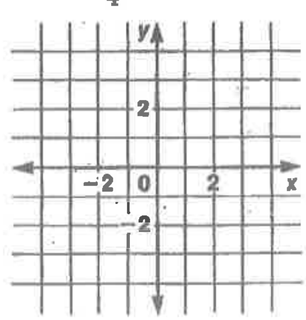
4. $y = x - 4$



5. $y = 3x + (-5)$



6. $y = \frac{1}{4}x$



The Pythagorean Theorem

One of the most famous theorems in the history of mathematics is the **Pythagorean Theorem.** It has to do with the sides of right triangles:

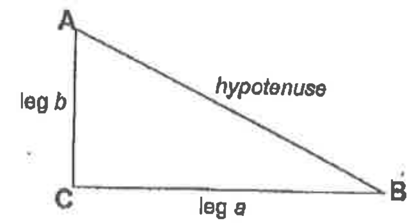
The Pythagorean Theorem

In any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

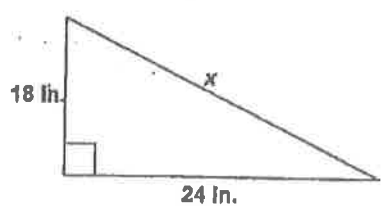
As a formula, the Pythagorean Theorem is:

$$a^2 + b^2 = c^2$$

You will often use this formula to solve problems.

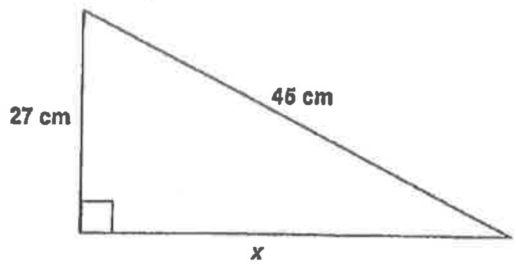


1. What is x?



a. 12 in.
b. 30 in.
c. 36 in.
d. 40 in.

2. What is x?



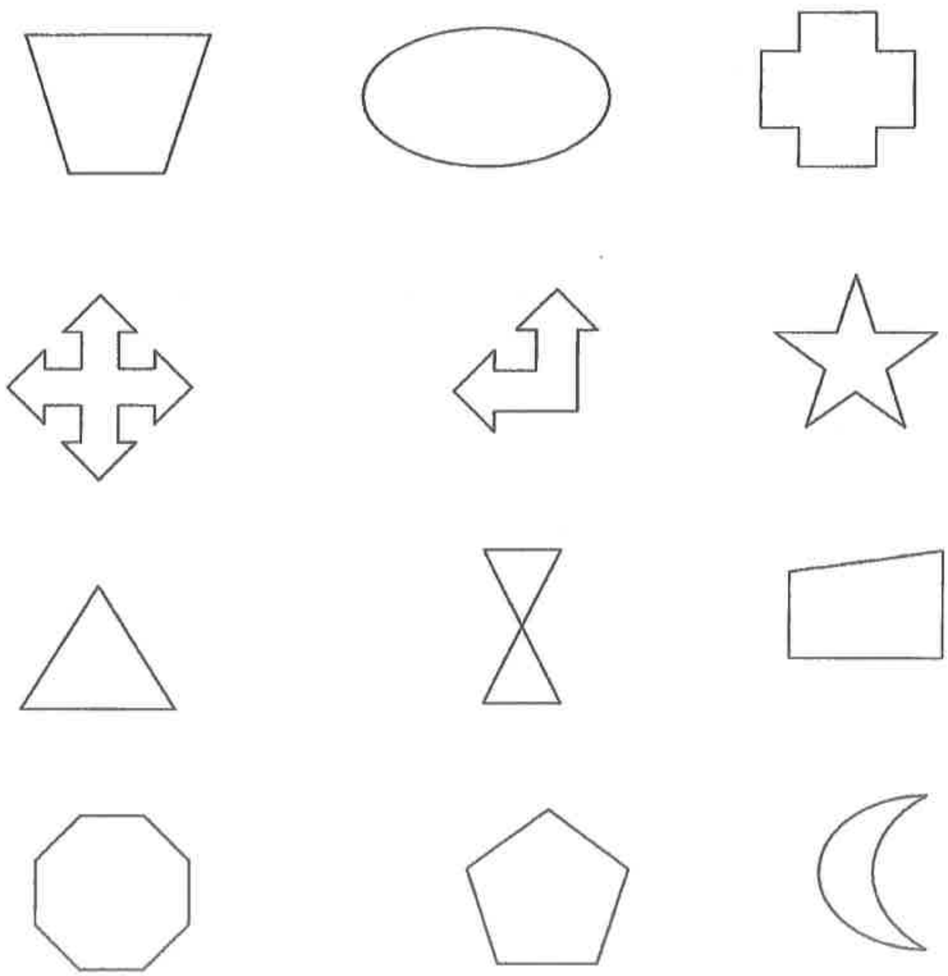
a. 20 cm
b. 25 cm
c. 28 cm
d. 36 cm

3. The length and width of a rectangle are 12 m and 5 m. What is the length of the diagonal? Show your work.

ROTATIONAL SYMMETRY

A shape has rotational symmetry if it fits onto itself two or more times in one turn.
The order of rotational symmetry is the number of times the shape fits onto itself in one turn.
A 2D shape has a line of symmetry if the line divides the shape into two halves – one being the mirror image of the other.

Write the order of rotational symmetry under each shape & letter. Also draw dotted lines to indicate lines of symmetry.



M A T H S

Order of Operations

$5(2)+7\bullet 3$

$$\frac{5^2-2^4}{3}$$

$$\frac{(6+8)(7-3)}{-2+3\bullet 4}$$

$19-5(-2)$

$-4(5-6)+3$

$$5^2-3\left(\frac{2}{3}+1\right)$$

The test scores for 20 students in a Spanish class are shown in the frequency table below.

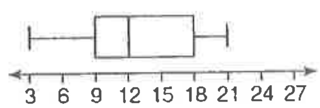
Interval	Frequency
90-99	4
80-89	3
70-79	8
60-69	4
50-59	1

According to the information shown, how many students received a score greater than a 69?

The median lies in which interval of the frequency table shown?

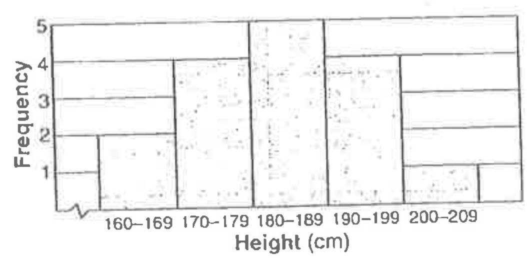
The upper quartile lies in which interval of the frequency table shown?

Which of the following sets of data values could represent the box-and-whisker plot below?



A) 3, 10, 11, 13, 21
B) 3, 6, 9, 12, 15, 18, 21
C) 3, 9, 10, 12,16, 18, 21
D) 3, 9, 10, 11, 13, 15, 18, 21

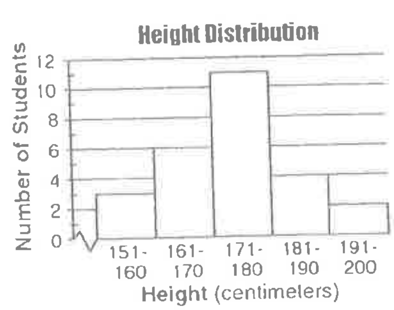
The accompanying histogram shows the heights of the students in Kyra's health class.



What is the total number of students in the class?

A) 15 B) 209 C) 16 D) 5

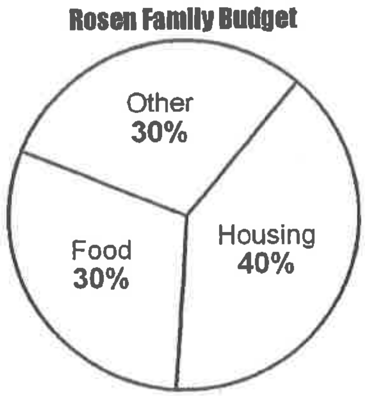
The accompanying histogram shows the height distribution for students in a high school mathematics class.



What is the total number of students in the class?

A) 28 B) 26 C) 49 D) 11

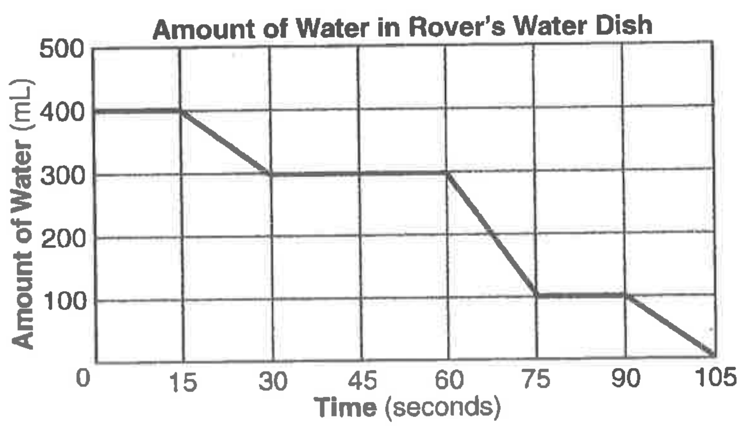
The Statistical Data Bureau published an analysis of incomes and expenditures of 100 average families throughout the United States. The circle graph below represents the Rosen family's monthly budget.



If their total monthly income is $1,820, how much money do they spend each month on food?

A) $546 B) $728 C) $606 D) $182

The accompanying graph shows the amount of water left in Rover's water dish over a period of time.



How long did Rover wait from the end of his first drink to the start of his second drink of water?

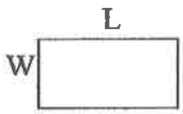
A) 60 sec B) 30 sec C) 10 sec D) 75 sec

Janae's first seven French grades for the year are 91, 87, 80, 99, 85, 78, and 90. What grade is at the 75th percentile?

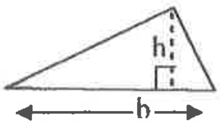
A) 90 B) 78 C) 90.5 D) 91

Formula Sheet

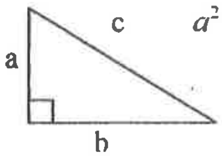
Plane Shapes



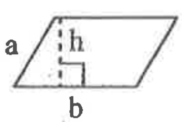
A = LW
P = 2L + 2W



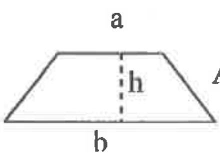
$A = \frac{1}{2}bh$



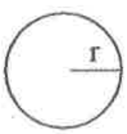
$a^2 + b^2 = c^2$



A = bh
P = 2a + 2b



$A = \frac{1}{2}h(a+b)$

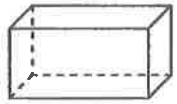


$C = 2\pi r$ or πd
$A = \pi r^2$

Angle Sum

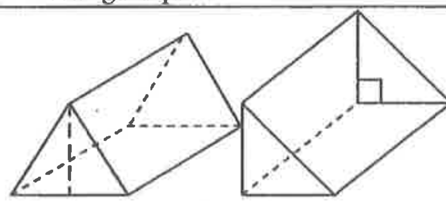
Sum of the angle measures = $180(n-2)$, where n equals the number of sides.

Space Shapes



Rectangular prism

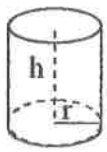
Total $SA = 2LW + 2LH + 2HW$
$LA = (\textit{perimeter of the base}) * H$
$V = LWH$



Triangular prisms

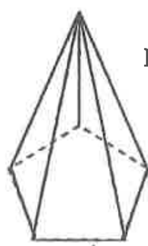
***H* = height of the prism**
***B* = area of the base**

Total $SA = 2B + LA$
$LA = (\textit{perimeter of the base}) * H$
$V = B * H$

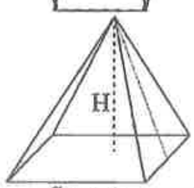


Cylinder

Total $SA = 2\pi r^2 + 2\pi r * h$
$LA = 2\pi r * h$
$V = \pi r^2 * h$



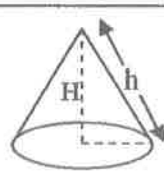
Pentagonal pyramid



Square pyramid

***H* = height of the pyramid**
***B* = area of the base**

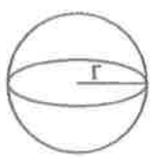
$V = \frac{1}{3}B * H$



Cone

Total $SA = \pi r^2 + \pi r h$

$V = \frac{1}{3}\pi r^2 H$



Sphere

Total $SA = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

Angle Sum Formula

The sum of the angle measures in a polygon can be found by the following formula:

Sum of angle measures = 180(n – 2)

where n is the number of sides on the polygon

Examples:

Find the sum of the angle measures in a square.
A square has four sides, so n = 4.
Sum = 180(n – 2)
Sum = 180(4 – 2)
Sum =180(2)
Sum =360°

Find the number of sides in a polygon that has an angle sum of 1080.°
Sum = 180(n – 2)
1080 = 180(n – 2)
If n is 8, the sides are balanced at 1080° so there are 8 sides.

Find the measure of each angle in a regular pentagon?
A pentagon has five sides, so n = 5.
Sum = 180(n – 2)
Sum = 180(5 – 2)
Sum =180(3)
Sum =540°
So, to find the measure of one angle, we need to divide the sum by the number of sides.
One angle = 540/5 = 108°

Practice Problems

Write the angle sum formula and show all work on a piece of loose-leaf paper.

1. Find the angle sum of a hexagon (6 sides).
2. Find the measure of one angle in a regular octagon (8 sides).
3. How many sides are there in a polygon with an angle sum of 720°?
4. Find the angle sum of a polygon with 23 sides.
5. Find the measure of one angle in a regular polygon with 14 sides.
6. How many sides are there in a polygon with an angle sum of 1,980°?
7. Find the angle sum of a polygon with 9 sides.
8. Find the measure of one angle in a regular polygon with 12 sides.

Directions: Draw and label a sketch of the space shape. Then, determine answer.

A right circular cone with a radius of 2.7 mm and a height of 30 mm. Find volume.	*A sphere with a radius of 4 cm. Find volume.*
A sphere with a diameter of 6 in. Find volume.	*A right circular cone with a diameter of 4 in. and a height of 20 in. Find volume.*
A square pyramid with base edges of 6 mm. and a height of 10 mm. Find volume.	*A triangular prism with base edges of 8 in. and a height of 12 in. Find the surface area and the volume.*
A triangular pyramid with base edges of 8 in. and a height of 12 in. Find volume. Can you use answer from triangular prism problem as a short-cut?	*A cylinder with 5 cm. radius on base and 10 cm. height. Find surface area and volume.*

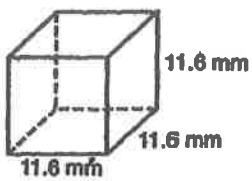
NAME ______________________________ DATE ______________

12-5 Practice
Surface Area: Prisms and Cylinders

Student Edition
Pages 632–637

Find the surface area of each solid. Round to the nearest tenth.

1.



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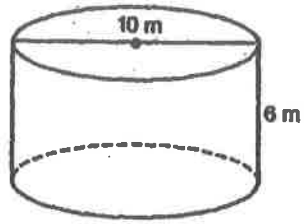

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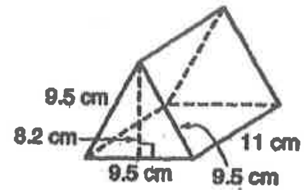

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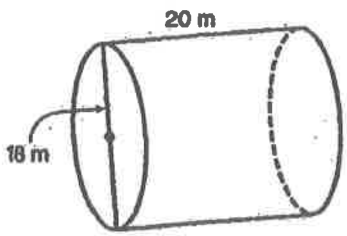

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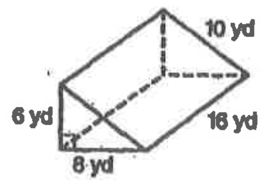

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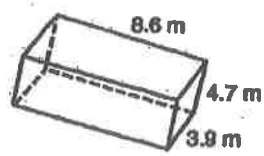

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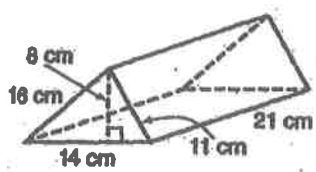

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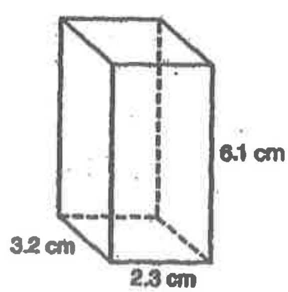

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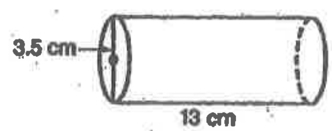

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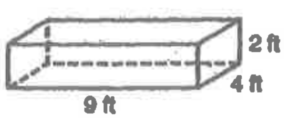

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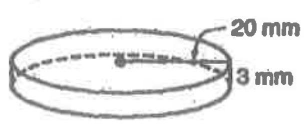

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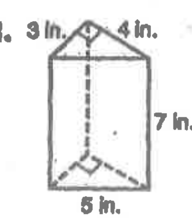


EXPONENTIAL GROWTH AND DECAY

GRAPHS OF EXPONENTIAL FUNCTIONS

An exponential function is an equation of the form $y = ab^x$ (with $b \geq 0$).

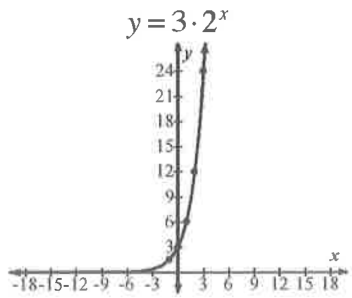
In many cases "a" represents a starting or initial value, "b" represents the multiplier or growth/decay factor, and "x" represents the time.

Example 1 Graph $y = 3 \cdot 2^x$

Make a table of values.

x	–1	0	1	2	3
y	1.5	3	6	12	24

Plot the points and connect them to form a smooth curve.



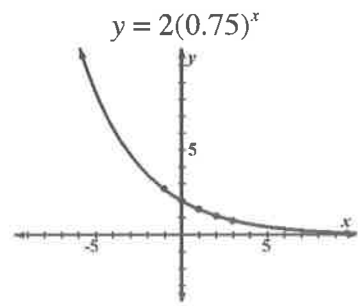
This is called an **increasing** exponential curve.

Example 2 Graph $y = 2(0.75)^x$

Make a table of values using a calculator.

x	–1	0	1	2	3
y	2.7	2	1.5	1.1	0.8

Plot the points and connect them to form a smooth curve.



This is called a **decreasing** exponential curve.

Problems

Make a table of values and draw a graph of each exponential function.

1. $y = 4(0.5)^x$
2. $y = 2(3)^x$
3. $y = 5(1.2)^x$
4. $y = 10(\frac{2}{3})^x$

8. A Honda Accord depreciates at 15% per year. Six year ago it was purchased for $21,000. What is it worth now?

9. Inflation is at a rate of 7% per year. Today Janelle's favorite bread costs $3.79. What would it have cost ten years ago?

10. Ryan's motorcycle is now worth $2500. It has decreased in value 12% each year since it was purchased. If he bought it four years ago, what did it cost new?

11. The cost of a High Definition television now averages $1200, but the cost is decreasing about 15% per year. In how many years will the cost be under $500?

12. A two-bedroom house in Nashville is worth $110,000. If it appreciates at 2.5% per year, when will it be worth $200,000?

13. Last year the principal's car was worth $28,000. Next year it will be worth $25, 270. What is the annual rate of depreciation? What is the car worth now?

14. A concert has been sold out for weeks, and as the date of the concert draws closer, the price of the ticket increases. The cost of a pair of tickets was $150 yesterday and is $162 today. Assuming that the cost continues to increase at this rate:

   a. What is the daily rate of increase? What is the multiplier?

   b. What will be the cost one week from now, the day before the concert?

   c. What was the cost two weeks ago?

15. AN APPLICATION: CHOOSING A CAR

Most cars decrease in value after you leave the dealer. However, some cars are now considered "classics" and actually increase in value. You have the choice of owning two cars: A 2006 Mazda Maita which is worth $19,000 but is depreciating 10% per year, or a classic 1970 Ford Mustang which is worth $11,500 and is increasing in value by 6% each year. Your tasks:

   a. Write an equation to represent the value of each car over time.

   b. Create tables and draw a graph to represent the value of each car for ten years on the same set of axes.

   c. Use your graph to determine approximately when the Mazda and the Ford have the same value.

SOLVING EXPONENTIAL GROWTH AND DECAY PROBLEMS

Example 1

Movie tickets now average \$9.75 a ticket, but are increasing 15% per year. How much will they cost 5 years from now?

The equation to use is: $y = ab^x$. The initial value $a = 9.75$. The multiplier b is always found by adding the percent increase (as a decimal) to the number "one," so $b = 1 + 0.15 = 1.15$. The time is $x = 5$. Substituting into the equation and using a calculator for the calculations: $y = ab^x = 9.75(1.15)^5 \approx 19.61$. In five years movie tickets will average about \$19.61.

Example 2

A powerful computer is purchased for \$2000, but on the average loses 20% of its value each year. How much will it be worth 4 years from now?

The equation to use is: $y = ab^x$. The initial value $a = 2000$. In this case the value is decreasing so multiplier b is always found by subtracting the percent decrease from the number "one," so $b = 1 - 0.2 = 0.8$. The time is $x = 4$. Substituting into the equation and using a calculator for the calculations: $y = ab^x = 2000(0.8)^4 = 819.2$. In four years the computer will only be worth \$819.20.

Example 3

Dinner at your grandfather's favorite restaurant now costs \$25.25 and has been increasing steadily at 4% per year. How much did it cost 35 **years ago** when he was courting your grandmother?

The equation is the same as above and $a = 25.25$, $b = 1.04$, but since we want to go back in time, $x = -35$. **A common mistake is to think that** $b = 0.96$. The equation is $y = ab^x = 25.25(1.04)^{-35} \approx 6.40$.

Example 4

If a gallon of milk costs \$3 now and the price is increasing 10% per year, how long before milk costs \$10 a gallon?

In this case we know the starting value $a = 3$, the multiplier $b = 1.1$, the final value $y = 10$, but not the time x. Substituting into the equation we get $3(1.1)^x = 10$. To solve this, you will probably need to guess and check with your calculator. Doing so yields $x \approx 12.6$ years. In Algebra 2 you will learn to solve these equations without guess and check.

Problems

5. The number of bacteria present in a colony is 180 at 12 noon and the bacteria grows at a rate of 22% per hour. How many will be present at 8 p.m.?

6. A house purchased for \$226,000 has lost 4% of its value each year for the past five years. What is it worth now?

7. A 1970 comic book has appreciated 10% per year and originally sold for \$0.35. What will it be worth in 2010?