Linear and Angular Velocity Examples

Example 1
Determine the angular displacement in radians of 6.5 revolutions.
Round to the nearest tenth.

Each revolution equals 2π radians. For 6.5 revolutions, the number of radians is 6.5 \times 2\pi or 13\pi. 13\pi radians equals about 40.8 radians.

Example 2
Determine the angular velocity if 4.8 revolutions are completed in 4 seconds.
Round to the nearest tenth.

The angular displacement is 4.8 \times 2\pi or 9.6\pi radians.

\[
\omega = \frac{\theta}{t}
\]

\[
\omega = \frac{9.6\pi}{4} \quad \theta = 9.6\pi, \ t = 4
\]

\[
\omega \approx 7.539822369 \quad Use \ a \ calculator.
\]

The angular velocity is about 7.5 radians per second.

Example 3
AMUSEMENT PARK
Jack climbs on a horse that is 12 feet from the center of a merry-go-round.
The merry-go-round makes \(3\frac{1}{4}\) rotations per minute. Determine Jack’s angular velocity in radians per second. Round to the nearest hundredth.

The merry-go-round makes \(3\frac{1}{4}\) or 3.25 revolutions per minute. Convert revolutions per minute to radians per second.

\[
\frac{3.25 \text{ revolutions}}{1 \text{ minute}} \cdot \frac{1 \text{ minute}}{60 \text{ seconds}} \cdot \frac{2\pi \text{ radians}}{1 \text{ revolution}} \approx 0.3403392041 \text{ radian per second}
\]

Jack has an angular velocity of about 0.34 radian per second.
Example 4
Determine the linear velocity of a point rotating at an angular velocity of $12\pi$ radians per second at a distance of 8 centimeters from the center of the rotating object. Round to the nearest tenth.

$$v = r\omega$$
$$v = (8)(12\pi) \quad r = 8, \, \omega = 12\pi$$
$$v \approx 301.5928947 \quad Use \ a \ calculator.$$  

The linear velocity is about 301.6 centimeters per second.

Example 5
Refer to the application in Example 3. Determine Jack’s linear velocity.

$$v = r\omega$$
$$v \approx (12)(0.34) \quad r = 12, \, \omega = 0.34$$
$$v \approx 4.08 \quad Use \ a \ calculator.$$  

Jack’s linear velocity is about 4.08 feet per second.

Example 6
BICYCLES  The tires on a bicycle have a diameter of 24 inches. If the tires are turning at a rate of 50 revolutions per minute, determine the bicycle’s speed in miles per hour (mph).

If the diameter is 24 inches, the radius is 12 inches. This measure needs to be written in miles. The rate needs to be written in hours.

$$v = \frac{r \omega}{1 \text{ rev} \left( \frac{2\pi}{1 \text{ rev} \left( \frac{60 \text{ min}}{1 \text{ h}} \right) \left( \frac{1 \text{ mi}}{5280 \text{ ft}} \right) \left( \frac{1 \text{ ft}}{12 \text{ in.}} \right) \right) \left( \frac{50 \text{ rev}}{1 \text{ min}} \right) \left( \frac{1 \text{ min}}{1 \text{ h}} \right)}$$

$$v \approx 3.569991652 \quad Use \ a \ calculator.$$  

The speed of the bicycle is about 3.6 miles per hour.